Special relativity theory part two

In part one we deliberately restricted to one space coordinate. Here we introduce a second space coordinate. Let's call the direction parallel to v the x_1-direction (x-direction for short) and the transverse direction the x_4-direction. Define $u_x = \Delta x/\Delta t$ and $u_4 = \Delta x_4/\Delta t$.

The velocity $u_x^* = \Delta x^*/\Delta t^*$ in another coordinate frame moving with velocity v is seen to be

$$u_x^* = \frac{u_x - v}{1 - u_x \cdot v / c^2} \qquad (15)$$

Using the Lorentz transformation for the time coordinate (9):

$$\Delta t^* = \frac{\Delta t - v \cdot \Delta x / c^2}{\sqrt{1 - v^2/c^2}}$$

we can write

$$\frac{\Delta t^*}{\Delta t} = \frac{1 - \left(\Delta x / \Delta t\right) \cdot v / c^2}{\sqrt{1 - v^2/c^2}}$$

and, using $u_x = \Delta x/\Delta t$,

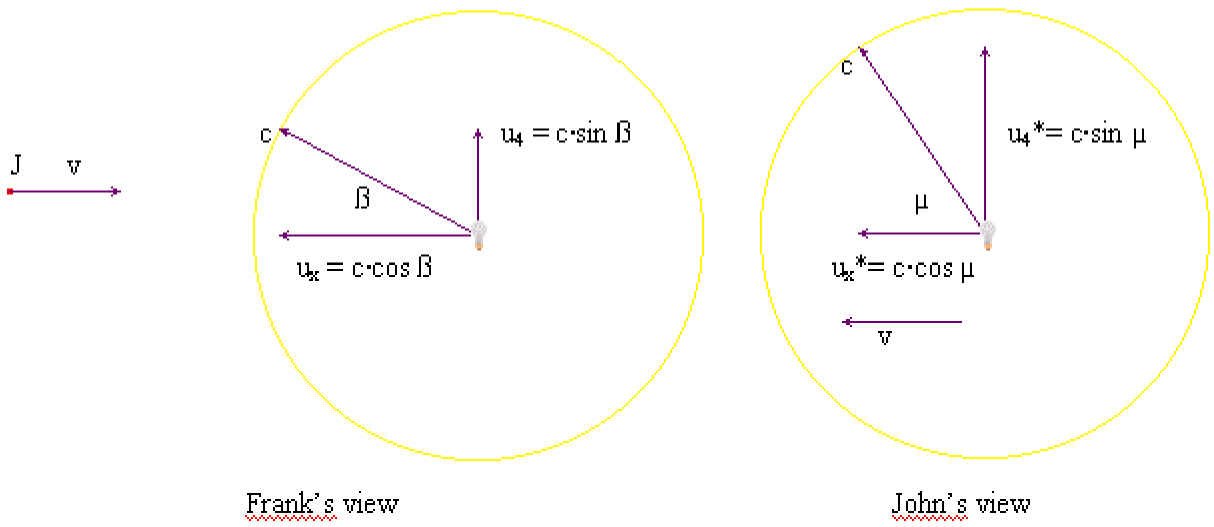
$$\frac{\Delta t^*}{\Delta t} = \frac{1 - u_x \cdot v / c^2}{\sqrt{1 - v^2/c^2}} \qquad (16)$$

Then we calculate the component normal to v, u_4^* ($\Delta x_4^* = \Delta x_4$ because the x_4-direction is normal to v):

$$u_4^* = \frac{\Delta x_4}{\Delta t^*} = \frac{\Delta x_4}{\Delta t} \cdot \frac{\Delta t}{\Delta t^*}$$

$$u_4^* = \frac{u_4 \cdot \sqrt{1 - v^2/c^2}}{1 - u_x \cdot v / c^2} \qquad (17)$$

Let's look at the emanation of light from a bulb in all directions in Frank's view and John's view



The magnitude of the velocity of light is always c. This remains true in any direction. Define the angle ß between the component of the velocity c and the direction of the relative velocity v of the bulb. Then the component u_x of c in the direction of v is u_x = c·cosß. Call the angle of u_x* with v µ. Then

u_x* = c·cos µ. From (15) we get

$$c \cdot \cos \mu = \frac{c \cdot \cos \beta - v}{1-(v/c)\cdot \cos \beta} \tag{18}$$

Dividing by c:

$$\cos \mu = \frac{\cos \beta - v/c}{1-(v/c)\cdot \cos \beta} \tag{19}$$

In the x_4-direction u_4 = c·sin ß and u_4* = c·sin µ. From (17) we get

$$c \cdot \sin \mu = \frac{c \cdot \sin \beta \cdot \sqrt{1-v^2/c^2}}{1-(v/c)\cdot \cos \beta} \tag{20}$$

Dividing by c:

$$\sin \mu = \frac{\sin \beta \cdot \sqrt{1-v^2/c^2}}{1-(v/c)\cdot \cos \beta} \tag{21}$$

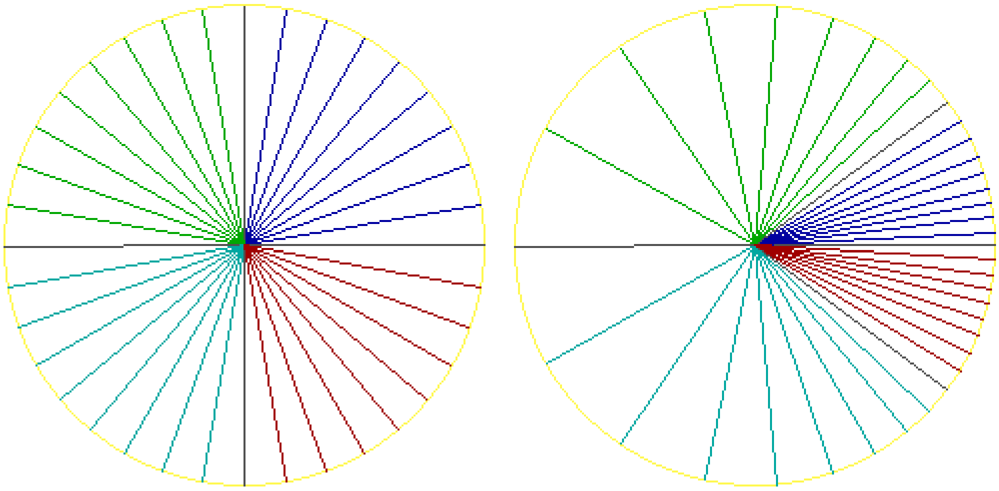
Dividing (21) by (19):

$$\tan \mu = \frac{\sin \beta \cdot \sqrt{1-v^2/c^2}}{\cos \beta - v/c} \tag{22}$$

The conclusion must be that, though the velocity of light is c in any direction, the direction itself does change on transforming to a frame with relative velocity v. This is called the aberration of light. This formula (22) is really a nice formula! We can find the expression for tan ß by changing v to –v and exchange ß and µ:

$$\tan\beta = \frac{\sin\mu \cdot \sqrt{1 - v^2/c^2}}{\cos\mu + v/c} \tag{23}$$

For v/c = 0.8 compare the rays with ß = 0°,10°,20°,..etc. to the transformed rays:



Frank's view (angle ß) John's view (angle µ)

How about the distance the light ray travels in different directions?

In Frank's view, the "ß-frame", this amounts to c·Δt. In John's view, "µ-frame", this depends on the position of the observer as the Doppler effect is different in different directions.We have

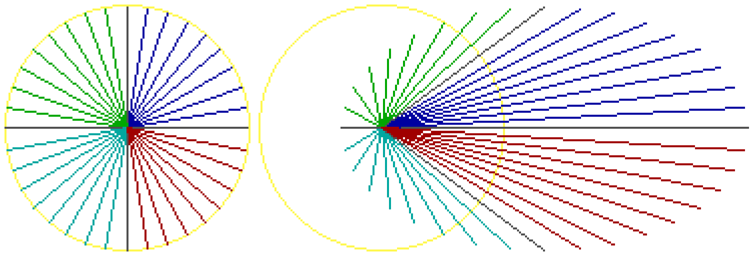
$$c \cdot \Delta t^* = \frac{c \cdot \Delta t \cdot \left(1 - u_x \cdot v/c^2\right)}{\sqrt{1 - v^2/c^2}} = \frac{c \cdot \Delta t \cdot \left(1 - (v/c) \cdot \cos\beta\right)}{\sqrt{1 - v^2/c^2}} \tag{24}$$

$$c \cdot \Delta t = \frac{c \cdot \Delta t^* \cdot \left(1 + u_x \cdot v/c^2\right)}{\sqrt{1 - v^2/c^2}} = \frac{c \cdot \Delta t^* \cdot \left(1 + (v/c) \cdot \cos\mu\right)}{\sqrt{1 - v^2/c^2}}$$

Or, equivalently

$$c \cdot \Delta t^* = \frac{c \cdot \Delta t \cdot \sqrt{1 - v^2/c^2}}{1 + (v/c) \cdot \cos\mu} \tag{25}$$

According to (22) we get the same angular distribution for µ. Let the yellow circle be the distance c·Δt:



Frank's view (angle ß) John's view (angle μ)

If cos ß = – cos μ (i.e. cos ß = {1 –√ (1 – v²/c²)}/(v/c)) then c·Δt* = c·Δt. In case of v/c = 0.8 this occurs for ß = 60° (u_x /c = cosß =1/2) and μ = 120° (u_x*/c = cos μ = –1/2).

Solving v, we calculate what velocity v the bulb must have to make a given Δt equal to Δt*:

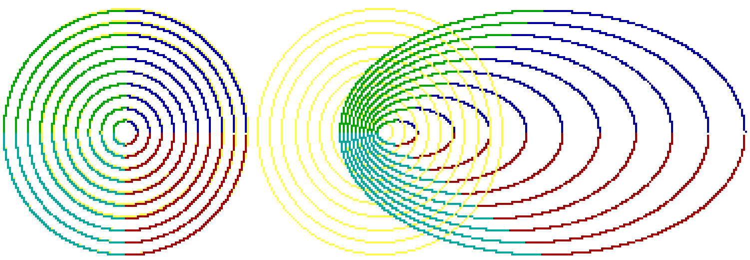
$$v = \frac{2u_x}{1 + u_x^2 / c^2}$$

or, as u_x = -u_x*,

$$v = \frac{-2u_x *}{1 + u_x *^2 / c^2}$$

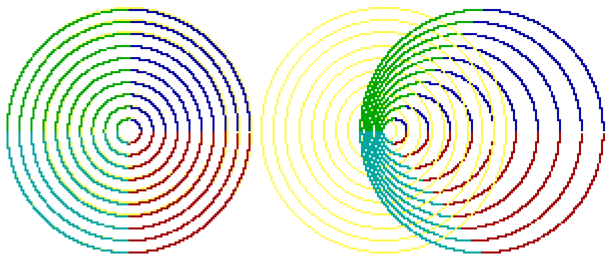
In the other cases one can see, on looking at the derivation in "Special relativity theory part one", that the Doppler effect on Δt causes elongation of the distance if cos ß < {1 –√ (1 – v²/c²)}/(v/c) or contraction if cos ß > {1 –√ (1 – v²/c²)}/(v/c).

Time dilation is the only cause of the elongation in the x_4-direction (transverse Doppler effect), while in the direction of v the effect of time dilation is accompanied by the longitudinal Doppler effect. The grey lines with ß = 90° and ß = 270° are elongated dividing by √ (1 – v²/c²) and rotated because of the aberration. For ß = 0° the factor (1–v/c) decreases the distance in that direction. For ß = 180° the factor (1+v/c) increases the distance in that direction. This gives the elliptic shape of the end points of the distances that are measured by observers that stand hand in hand in a circle around the emitting bulb. If Δt is n oscillation periods T the distances are a number of n wavelengths. This can be depicted as circles in Frank's view and ellipses in John's view representing wave tops with equal phase.

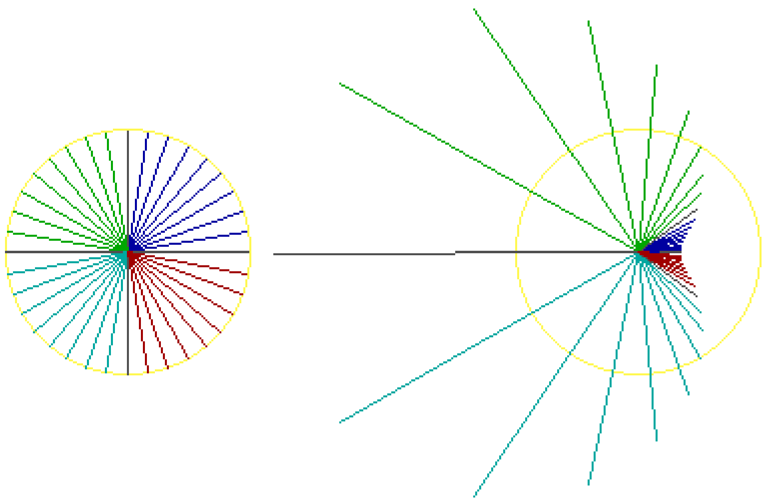


Frank’s view (angle ß) John’s view (angle μ)

Compare this to the classical non-relativistic result:



Because frequency f and wavelength λ have the relation f · λ = c , the observed frequencies in the different directions can be found by inversion in a circle. Then magnitude of frequency is proportional to the length of the lines coming from the position of the bulb. The yellow circle represents the rest frequency of the source:



angle ß angle μ

Recall that $u_x/c = \cos ß$, $u_x{*}/c = \cos \mu$, $u_4/c = \sin ß$, $u_4{*}/c = \sin \mu$ and using (21)

$$\sin \mu = \frac{\sin \beta \cdot \sqrt{1 - v^2 / c^2}}{1 - (v/c) \cdot \cos \beta}$$

gives

$$\sqrt{1 - u_x{*}^2 / c^2} = \frac{\sqrt{1 - u_x{}^2 / c^2} \cdot \sqrt{1 - v^2 / c^2}}{1 - u_x v / c^2}$$

and inverting

$$\frac{1}{\sqrt{1-u_x{*}^2/c^2}}=\frac{1}{\sqrt{1-u_x{}^2/c^2}}\frac{1-u_x v/c^2}{\sqrt{1-v^2/c^2}} \tag{26}$$

Obviously, $\frac{1}{\sqrt{1-u_x{}^2/c^2}}$ transforms as Δt.

We could have expected this result as, according to (16) and (17), $u_4/u_4{*}= \Delta t{*}/\Delta t$.

Multiplying left and right part by left and right part of (15) $u_x{*}=\frac{u_x-v}{1-u_x\cdot v/c^2}$ gives

$$\frac{u_x{*}}{\sqrt{1-u_x{*}^2/c^2}}=\frac{u_x-v}{\sqrt{1-u_x{}^2/c^2}}\frac{1}{\sqrt{1-v^2/c^2}} \tag{27}$$

Define

$$m=\frac{m_0}{\sqrt{1-u_x^2/c^2}} \quad \text{and} \quad p_x=m\cdot u_x=\frac{m_0\cdot u_x}{\sqrt{1-u_x^2/c^2}}$$

then multiplying (26) and (27) by m_0 gives

$$m{*}=\frac{m-v\cdot p_x/c^2}{\sqrt{1-v^2/c^2}} \tag{28}$$

$$p_x{*}=\frac{p_x-m\cdot v}{\sqrt{1-v^2/c^2}} \tag{29}$$

p_x and m transform as x and t.

Mark that $m=\frac{m_0}{\sqrt{1-u_x^2/c^2}}$ can be written as

$$m^2c^2 - p_x^2 = m_0^2c^2 \tag{30}$$

What can be the meaning of p_x and m and m_0?

At first we can identify m_0 as the value of m when $u_x = 0$ or m* when $u_x = v$. It's the value of m that is measured by an observer who measures a transverse direction of the velocity c.

In spite of the very suggestive symbols p and m there is no conclusive answer to the rest of the 3-fold question! If we were to proceed with electrodynamics for instance, then m could mean electric charge density and p electric current density. But first we will proceed with dynamics.

What about the medium in which the waves act?

We see that the pattern is different in John's and in Frank's view. The observation of the waves by John is identical in both views but the pattern of the waves is different. Looking at the pattern would be a simple way to determine if John is moving or the bulb with respect to the medium. But then the medium would have an observable existence and the observed pattern depend on its absolute state of motion. According to the relativity principle this is impossible and we conclude that the medium is unobservable. The conclusion that the medium is vacuum space (or space-time) cannot be proven. We will see that the medium is responsible for the interaction between the source and the observer. This seems very reasonable since the process of observation cannot be subject to observation itself. Light cannot be seen itself but is the means of interaction by which we see things.

In quantum theory there has been much discussion about the meaning of the wave function. The word function already points to a mathematical interpretation instead of real physical waves. Yet by means of the concept of "observable" that operates on the wave function and the amplitude of the wave, being interpreted as probability distribution for such observable quantities, reality is saved. However there is the alternative formulation of David Bohm which can give the same results as the conventional one. The advantage of the formulation of Bohm is a more realistic connection between particles and waves. It was considered a drawback for his way of looking at things that there appears a "quantum potential" that is responsible for a kind of overall interaction between particles that does not obey the rule of relativity theory that information should not run faster than c. But conventional quantum theory formulation equally suffers from these strange conditions that are however confirmed by experiments. Does God play dice or is there an underlying reality, to be discovered, that provides the missing information? In my opinion the process of observation (or measurement if you like) is the interacting medium that is itself unobservable. This process of observation models the medium in a way that the observable phenomena become apparent for observing devices. The Doppler effect provides us with a phenomenon of wave nature that depends on relative motion. Seen in this way it seems the right candidate to unite relativity and quantum mechanics.